Based on National Curriculum of Pakistan 2022-23

Textbook of

Physics

12

National Curriculum Council

Ministry of Federal Education and Professional Training

National Book Foundation
as
Federal Textbook Board
Islamabad

Government Approval
Approved by the National Curriculum Council (NCC), Ministry of Federal Education and Professional Training, Islamabad
vide letter No.F.1(2)/NCC-NOC/Phy/NBF-G12, dated April 09, 2025

A Textbook of Physic for Grade 12
based on National Curriculum of Pakistan (NCP) 2022-23

Authors
Naeem Nazeer (Managing Author)
Nazir Ahmed Malik, Ajaz Ahmad, Dr. Ejaz Ahmed, Aamir Ullah Khan, Ghulam Murtaza Sidddiqui, Dr. Humaira Anwar (Co-Authors)

Supervision of Curriculum and Review Process
Dr. Tabassum Naz
Joint Educational Advisor, National Curriculum Council (NCC)
Ministry of Federal Education and Professional Training, Government of Pakistan, Islamabad

NCC Review Committee Members
Mr. Nisar Khan, Mr. Sajid Raza Syed, Mr. Asad Raza
Desk Officer: Mrs. Zehra Khushal (Assistant Educational Advisor), National Curriculum Council (NCC)

Supervision of the FBISE Review Committee
Mr. Mirza Ali, Director (Test Development)

FBISE Review Committee Members
Mr. Muhammad Jahangir Mirza, Ms. Robina Ahmad, Mr. Muhammad Imran Khaliq, Ms. Sehrish Imran
Desk Officer: Syed Zulfiqar Shah, Deputy Secretary, Research and Acadamics

---

NBF Textbooks Development Supervision
Dr. Kamran Jahangir
Managing Director, National Book Foundation (NBF)

In-Charge, NBF Textbooks Development
Mansoor Ahmad, (Deputy Director)

TEST EDITION

Printed in Pakistan
First Edition: First Impression: June 2025 | Pages: 230 | Quantity: 100000

Price: PKR 415/-, **Code:** STE-729, **ISBN:** 978-969-37-1830-0
Printer: Zarina Shahab Printers, Lahore

For details on additional publications from the National Book Foundation, please visit our website at www.nbf.org.pk
You can also reach us by phone at 051 9261125 or via email at books@nbf.org.pk
For feedback or corrections, kindly send your comments to 'nbftextbooks@gmail.com' and 'textbooks@snc.gov.pk'

Note
All illustrations, artwork, and images in this book are intended solely for educational and promotional purposes, benefiting the public interest.

Preface

This Textbook for Physics Grade 12 has been developed by NBF according to the National Curriculum of Pakistan 2022-2023. The aim of this textbook is to enhance learning abilities through inculcation of logical thinking in learners, and to develop higher order thinking processes by systematically building the foundation of learning from the previous grades. A key emphasis of the present textbook is creating real life linkage of the concepts and methods introduced. This approach was devised with the intent of enabling students to solve daily life problems as they grow up in the learning curve and also to fully grasp the conceptual basis that will be built in subsequent grades.

After amalgamation of the efforts of experts and experienced authors, this book was reviewed and finalized after extensive reviews by professional educationists. Efforts were made to make the contents student friendly and to develop the concepts in interesting ways.

The National Book Foundation is always striving for improvement in the quality of its textbooks. The present textbook features an improved design, better illustration and interesting activities relating to real life to make it attractive for young learners. However, there is always room for improvement, the suggestions and feedback of students, teachers and the community are most welcome for further enriching the subsequent editions of this textbook.

May Allah guide and help us (Ameen).

Dr. Kamran Jahangir
Managing Director

Practical Applications of Physics-XII in Everyday Life

Welcome to the fascinating world of physics! This book is designed to take you on a journey through the fundamental principles and concepts that govern our universe. From the intricacies of gravitational potential to the mysteries of quantum physics, each unit in this book will reveal the beauty and relevance of physics in our daily lives.

As you explore the world of physics, you'll discover the numerous career paths available to you. Whether you're interested in pursuing a career in research, engineering, medicine, or technology, physics will provide you with a solid foundation for success.

Let's take a glimpse into the exciting world of physics that awaits you:

In **Unit 15, Gravitation**, you'll discover how the concept of gravitational potential energy is crucial in designing roller coasters, understanding the motion of planets, and even predicting the trajectory of spacecraft. Careers in aerospace engineering, astrophysics, and geophysics rely heavily on this concept.

Unit 16, Statistical Mechanics and Thermodynamics, will show you how the principles of thermodynamics govern and impacting our daily lives in countless ways. Careers in mechanical engineering, chemical engineering, and materials science rely on a deep understanding of thermodynamics.

In **Unit 17, Simple Harmonic Motion**, you'll explore how this fundamental concept is used in designing medical equipment, such as MRI machines, and musical instruments, like guitars and violins. Careers in biomedical engineering, mechanical engineering, sound and music technology rely on an understanding of simple harmonic motion.

Unit 18, Diffraction and Interference, will reveal the secrets and the behavior of waves in different phenomenon. Careers in photonics, optics, and acoustics rely heavily on this concept.

Unit 19, Electric Potential and Capacitor, will introduce you to the world of energy storage and transmission, crucial for understanding how batteries, capacitors, and electrical grids work. Careers in electrical engineering, renewable energy, and energy storage rely on a deep understanding of electric potential and capacitors.

In **Unit 20, Alternating Current**, you'll discover how AC circuits power our homes, industries, and technologies, and learn about the innovative applications of AC in medical equipment and transportation systems. Careers in electrical engineering, power engineering, and telecommunications rely heavily on this concept.

Unit 21, Quantum Physics, will take you on a journey into the fascinating realm of the tiny, where the principles of quantum mechanics govern the behavior of atoms, molecules, and subatomic particles. Careers in materials science, nanotechnology, and quantum computing rely on a deep understanding of quantum physics.

Unit 22, Nuclear Physics, will explore the mysteries of nuclear reactions, radioactivity, and the applications of nuclear energy in medicine, industry, and power generation. Careers in nuclear engineering, medical physics, and radiation therapy rely heavily on this concept.

In **Unit 23, Cosmology,** you'll embark on a cosmic journey to explore the origins, evolution, and fate of our universe, delving into the mysteries of dark matter, dark energy, and the expansion of the cosmos. Careers in astrophysics, cosmology, and space exploration rely on a deep understanding of cosmological principles.

Unit 24, Earth's Climate, will examine the complex relationships between our planet's atmosphere, oceans, and land surfaces, highlighting the impact of human activities on climate change and the importance of sustainable practices. Careers in environmental science, climate modeling, and sustainability rely heavily on this concept.

In **Unit 25, Medical Imaging,** you'll discover how physics principles, such as X-ray computed tomography (CT) scans, magnetic resonance imaging (MRI), and positron emission tomography (PET) scans, have revolutionized medical diagnosis and treatment. Careers in medical physics, biomedical engineering, and radiology rely on a deep understanding the principles of medical imaging.

Finally, **Unit 26, Nature of Science: A Debate,** will challenge you to think critically about the scientific method, the role of experimentation and observation, and the ethics of scientific inquiry. Careers in science policy, science communication, and science education rely heavily on a deep understanding of the nature of science.

As you embark on this physics journey, remember that the concepts and principles you'll learn are not just abstract ideas - they have a profound impact on our daily lives, from the technology we use to the environment we inhabit. By learning these fundamental physics concepts, you'll gain a deeper understanding of the world around you and develop problem-solving skills essential for innovative careers in science, technology, engineering, and mathematics (STEM).

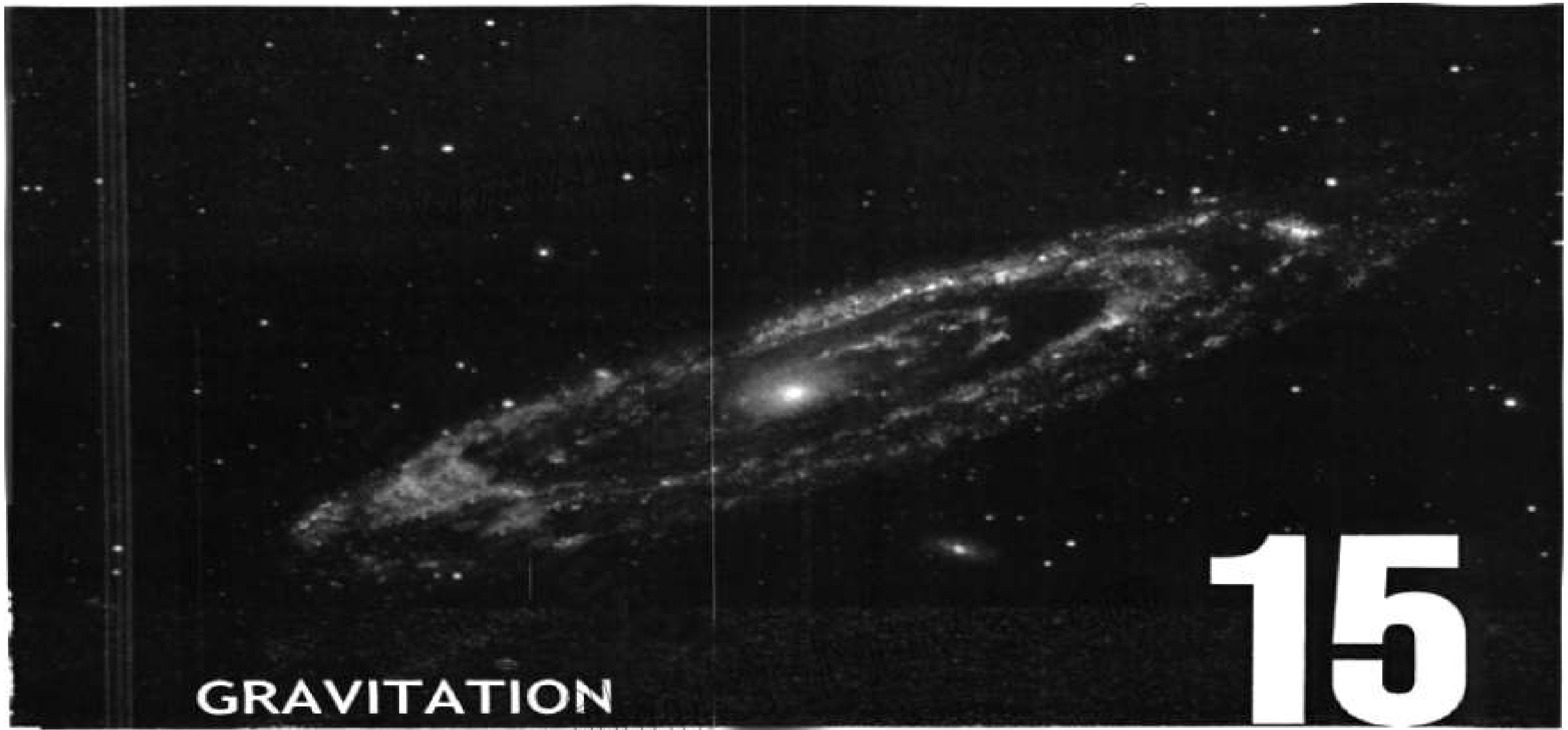
Some potential career paths in physics include:

- Research scientist
- Medical physicist
- Biomedical engineer
- Materials scientist
- Nanotechnologist
- Environmental scientist
- Climate modeler
- Sustainability specialist
- Science policy analyst
- Quantum computing specialist
- Science communicator
- Science educator
- Engineer (mechanical, electrical, aerospace, etc.)

These are just a few examples of the many exciting career paths available to physics students. Get ready to explore, discover, and be amazed by the wonders of physics!

Managing Author
Physics-XII

Contents



15 GRAVITATION

Student Learning Outcomes (SLOs)

The student will

- Define and calculate gravitational field strength [this will include more challenging problems than in Grade-9. It will involve use of $g = \frac{GM}{r^2}$]
- Analyse gravitational fields by means of field lines. This includes knowing that for a point outside a uniform sphere, the mass of a sphere may be considered to be a point mass at its center.]
- Apply Newton's law of gravitation to solve problems [$F = G\frac{m_1 m_2}{r^2}$ for the force between two-point masses to solve problems].
- Analyze circular orbits in gravitational fields [By relating the gravitational force to the centripetal acceleration it causes]
- Analyze the motion of geostationary satellites [This includes knowing that a geostationary orbit remains at the same point above the Earth's surface, with an orbital period of 24 hours, orbiting from west to east, directly above the Equator].
- Derive the equation for gravitational field strength [From Newton's law of gravitation and the definition of gravitational field, the equation $g = \frac{GM}{r^2}$ for the gravitational field strength due to a point mass].
- Analyse why g is approximately constant for small changes in height near the Earth's surface.
- Define and calculate gravitational potential [Use $\phi = \frac{-GM}{r}$ for the gravitational potential in the field due to a point mass] [At a point as the work done per unit mass in bringing a small test mass from infinity to the point]
- Justify how the concept of gravitational potential leads to the gravitational potential energy of two-point masses [Use $E_p = -G\frac{Mm}{r}$ in problems is expected]

Gravity, a force that has fascinated scientists for centuries, is essential in shaping the universe. It keeps us firmly held on Earth and controls the movements of stars in galaxies. Newton's law of gravitation has been crucial in helping us comprehend this force.

Our galaxy, the Milky-Way, is a prime example of gravity's strength. It is a vast disk of stars, dust, and gas held together by the powerful gravitational force of its center. Even though we are located near the edge of the galaxy, about 26,000 light-years from the center, we are not alone. The Milky-Way is part of the Local Group, a cluster of galaxies that includes the Andromeda Galaxy, just 2.5 million light-years away. Gravity is the master of this cosmic dance. It controls the movements of stars within our galaxy and influences how galaxies interact with each other. As we explore gravity further, we develop a deeper understanding of the force that shapes the universe on large scale.

15.1 NEWTON'S LAW OF UNIVERSAL GRAVITATION

Newton formulated a law called the law of universal gravitation to describe the force of attraction between various objects in the universe; which is stated as follows:

Every object in the universe attracts every other object with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

Newton's law of universal gravitation revolutionized our understanding of the physical world and provided a mathematical framework for explaining the motion of celestial bodies such as planets, moons, and stars. It laid the foundation for the development of classical mechanics and helped to explain many phenomena in the natural world, from the tides caused by the gravitational pull of the moon to the orbiting of planets around the sun.

Today, Newton's law of universal gravitation is still used as a fundamental principle in physics and astronomy, although it has been refined and expanded upon by later theories such as Einstein's theory of general relativity. Nevertheless, Newton's law of universal gravitation remains a cornerstone of our understanding of the force that governs the motion of objects in the universe.

Consider two spherical bodies of masses 'm_1' and 'm_2' separated by distance 'r', as shown in Fig. 15.1. By definition of Newton's law of universal gravitation, the force of gravity 'F_g' is:

$$F_g \propto m_1 \times m_2$$

$$F_g \propto \frac{1}{r^2}$$

Combining both relations, we get:

$$F_g \propto \frac{m_1 \times m_2}{r^2}$$

Replacing proportionality with constant, we get:

$$F_g = G\frac{m_1 \times m_2}{r^2}$$

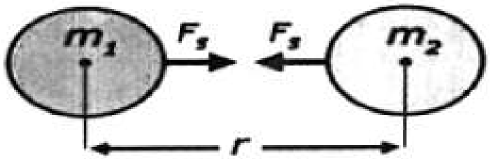


Figure 15.1: Illustration of gravitational force between two objects.

Where 'G' is constant of proportionality and is known as gravitational constant. Its value is 6.67×10^{-11} N m^2 kg^{-2}.

Gravitational force is always attractive and does not depend upon the medium between the masses. The gravitational force is an inverse-square force: it decreases by a factor of 4 when the distance increases by a factor of 2, it decreases by a factor of 9 when the distance increases by a factor of 3, and so on. Graph in Fig. 15.2 is a plot of the magnitude of the gravitational force as a function of the distance, between two objects.

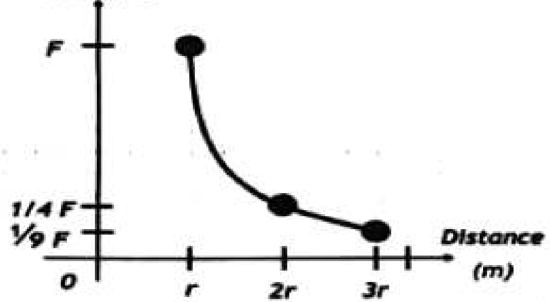


Figure 15.2: Variation of the gravitational force as a function of the distance.

According to Newton's law of universal gravitation, all objects attract each other. So, if the force on mass 'm_2' caused by mass 'm_1' is 'F_{21}', there is also a force on mass 'm_1' caused by mass 'm_2', which is 'F_{12}'. These forces are equal in magnitude but opposite in direction. Therefore, we can conclude that the forces acting on two objects due to gravitational force demonstrate action and reaction making a pair, as shown in the Fig. 15.3.

Figure 15.3: Illustration of gravitational action reaction pair.

The forces 'F_{12}' and 'F_{21}' can be mathematically equated as:

$$F_{12} = -F_{21}$$

These two forces act like action and reaction which are equal to each other in magnitude but opposite in direction. It means that Newton's universal law of gravitation is consistent with Newton's 3rd law motion.

Example 15.1: Two bodies A and B are placed at a distance of 1 m from each other. What will be the force of attraction between them if their masses are 45 kg and 50 kg respectively?

Given: r = 1 m $\quad m_1 = 45$ kg $\quad m_1 = 50$ kg

To Find: $F_g = ?$

Solution: Using Newton's law of universal gravitation: $\quad F_g = G\dfrac{m_1 m_2}{r^2}$

Putting values: $\quad F_g = \dfrac{6.673\times10^{-11}\times45\times50}{(1)^2}$

Hence $\quad F_g = 1.5\times10^{-7}\,N$

This implies that we are drawn towards each other, but the force is so minuscule, around 10^{-7} N, that it goes unnoticed unless we employ highly sensitive instruments. Even massive objects such as ships and buildings experience a very slight gravitational pull.

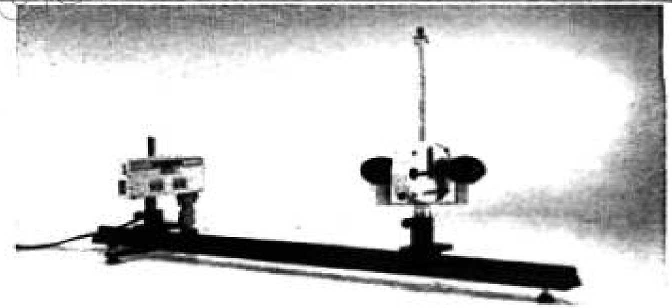
Assignment 15.1

1) The mass of Earth is 6×10^{24} kg and that of the moon is 7.4×10^{22} kg, with a distance of 3.84×10^5 km between them. Calculate the force exerted by the Earth on the moon.

2) Why do we say that law of gravitation is a universal law?

POINT TO PONDER

The gravitational constant 'G' is crucial for calculating gravitational forces accurately. However, measuring such a small number was a challenge. It took scientists 100 years after Newton's work to create a device capable of measuring G. In 1798, Henry Cavendish, an Englishman, used a torsion balance apparatus (shown in figure) to measure the gravitational force between two objects. This experiment not only helped determine G but also raised questions about whether gravitational force varies in different mediums, such as air and water. Knowing G also led to obtaining precise values for Earth's mass and other astronomical masses.



15.2 GRAVITATIONAL FIELD STRENGTH

The area around a massive object (like Earth, sun, etc.) where its gravitational force acts is known as gravitational field.

Gravitational field is represented by field lines that indicate the direction and strength of gravity, as shown in Fig. 15.4. Closer lines represent a stronger field, while farther apart lines indicate a weaker field.

The gravitational field lines point radially inward uniformly in all directions for a point mass (a point mass is treated as if all its mass is concentrated at a single point in space), forming a symmetric pattern. Since a gravitational force is an interaction between a test mass and the gravitational field created by the source mass, therefore from Newton's second law of motion:

$$a = \frac{F}{m}$$

Similarly, $$a_g = \frac{F_g}{m} = g$$

Gravitational field can be defined by gravitational field strength, denoted by 'g'. The gravitational field strength is a vector with a magnitude of 'g' pointing in the direction of the gravitational force. The value of 'g' is the gravitational force on a unit mass at that point, or '$g = F_g/m$'. Therefore, gravitational field strength and acceleration due to gravity are equivalent.

Value of 'g' on Surface of the Earth

Newton's law of universal gravitation shows that the value of 'g' depends on mass and distance.

For example, consider an object (stone) of mass 'm_O' placed on surface of Earth. Let 'M_E' be the mass of the Earth and radius of Earth 'R_E' is the distance between their centres (as radius of stone is very small compared to radius of Earth, therefore it is ignored). The gravitational force between the stone and Earth is:

$$F_g = G\frac{m_O M_E}{R_E^2} \qquad \text{—— (15.1)}$$

The gravitational force F_g by Newton's second law is:

$$F_g = W = m_o g \qquad \text{\_\_\_\_\_\_ (15.2)}$$

Comparing Eq. (15.1) and Eq. (15.2), we get: $m_o g = G\dfrac{m_o M_E}{R_E^2}$

or $$g = G\frac{M_E}{R_E^2} \qquad \text{\_\_\_\_\_\_ (15.3)}$$

For Earth we know that mass of Earth $M_E = 6 \times 10^{24}$ kg and radius of Earth $R_E = 6.4 \times 10^6$ m and $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, putting these values in Eq. (15.4), we get:

$$g = 6.67 \times 10^{-11} \times \frac{6 \times 10^{24}}{(6.4 \times 10^6)^2} \quad \text{or} \quad g = 9.77 \text{ m s}^{-2} = 9.8 \text{ m s}^{-2}$$

Eq. (15.3) shows that the value of 'g' does not depend upon the mass 'm_o' of the body. This means that light and heavy bodies should fall at same rate. This equation also shows that gravitational field strength depends only on mass of Earth 'M_E' and radius of Earth 'R_E'. Therefore, on any other planet's surface, both the value of 'g' and our weight will be influenced by the planet's mass 'm' and radius 'r', therefore the Eq. (15.3) in more general form can be expressed as:

$$g = G\frac{m}{r^2} \qquad \text{\_\_\_\_\_\_ (15.4)}$$

Variation of 'g' with Altitude

Moving away from surface of Earth may change 'g' and therefore our weight. The value of 'g' at a given place depends upon the distance from the centre of Earth, as shown in Fig. 15.4.

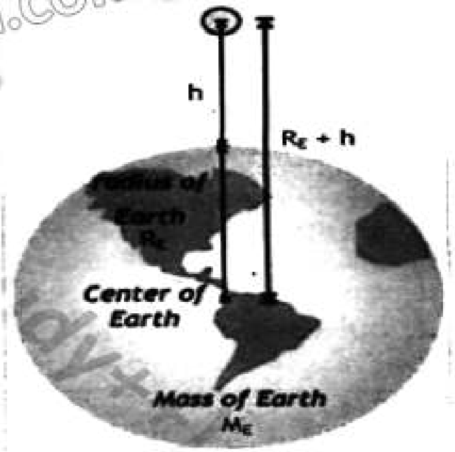


Figure 15.4: Variation of 'g' with altitude.

Let 'g_h' be the value of acceleration due to gravity at a height 'h' from the surface of the Earth. We can rewrite gravitational field strength from Eq. (15.3) as:

$$g_h = G\frac{M_E}{(R_E + h)^2} \qquad \text{\_\_\_\_\_\_ (15.5)}$$

As, $$g = G\frac{M_E}{R_E^2}$$

or $$GM_E = gR_E^2 \qquad \text{\_\_\_\_\_\_ (15.6)}$$

Putting Eq. (15.6) in Eq. (15.5), we get:

$$g_h = \frac{gR_E^2}{(R_E + h)^2} \qquad \text{\_\_\_\_\_\_ (15.7)}$$

This equation shows that:

As we go further away from the centre or surface of the Earth, the value of 'g' decreases.

Table 15.1: Variation in Gravitational Acceleration with Altitude

Altitude (km)	g_h (m s^{-2})	Example
0	9.8	Average Earth radius
8.8	9.8	Mount Everest
36.6	9.7	Highest manned balloon
400	8.7	Space shuttle orbit
35,700	0.2	Communication satellites

The change in 'g' is only noticeable at very long distances, as indicated in the Table 15.1.

Example 15.2: Shooting stars are meteors that burn up in Earth's atmosphere. Meteors become visible between about 75 to 120 km above Earth. What is the gravitational field strength at 120 km above earth surface?

Solution: The gravitational field strength of Earth is given by:

$$g_h = \frac{gR_E^2}{(R_E + h)^2}$$

Putting values, we get: $g_h = \frac{9.8 \times (6.4 \times 10^6)^2}{(6.4 \times 10^6 + 0.12 \times 10^6)^2}$ or $g_h = 9.4 \text{ m s}^{-1}$

The value of 'g' at 120 km above Earth surface is slightly less than its value at surface of Earth.

Assignment 15.2

What will be the value of gravitational filed strength 'g' at 35,700 km, where geostationary satellites orbit around the Earth?

Gravitational Fields Lines

To better understand the gravitational field surrounding the Earth, we can visualize the planet as a perfectly uniform sphere. This simplification allows us to analyse the gravitational effects more easily. A uniform sphere has a consistent density throughout. The concept of a uniform sphere simplifies the understanding of complex mass distributions, which is crucial in astrophysics and planetary science. It allows for easier calculations of gravitational forces for stars, planets, and moons without considering differences in their density. Its gravitational field behaves as follows:

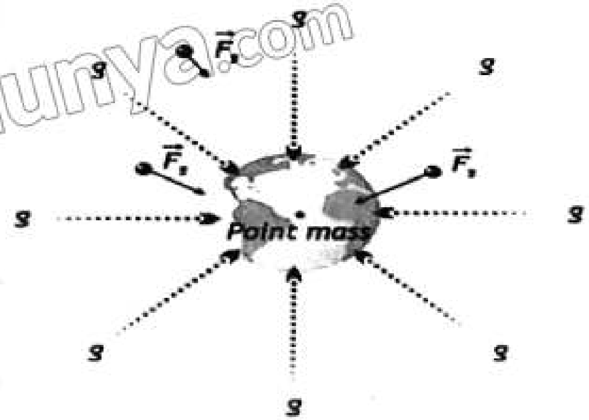


Figure 15.5: Gravitational Field Lines.

A. Outside the Sphere: The field acts as if all mass is concentrated at the centre, resembling that of a point mass with the same total mass.

For locations that are situated outside the Earth's surface, we can conceptualize the Earth's mass as being concentrated at a single point located at its centre. This model is useful because it simplifies the calculations involved in understanding gravitational forces.

As shown in Fig. 15.5, this representation illustrates how the strength of the gravitational field reduces rapidly as one moves away from the Earth. This behaviour aligns with our expectations as one would expect from the inverse-square nature of Newton's law of universal gravitation.

B. Inside the Sphere: The gravitational field strength increases steadily toward the surface, as only the mass within the radius affects the field at any point inside.

Example 15.3: The largest planet in our solar system is Jupiter, it has a mass of 1.898×10^{27} kg and radius as 7.15×10^{7} m. Determine the free-fall acceleration at the surface of Jupiter? Also calculate the weight of 60 kg astronaut on it.

Given: $m_J = 1.898\times10^{27}$ kg $r_J = 7.15\times10^{7}$ m Mass of astronaut = 60 kg

To Find: g_J = ? W_A = ?

Solution: The gravitational field strength of Jupitar is given by:

$$g_J = G\frac{m_J}{r_J^2}$$

Putting values, we get: $g_J = 6.67\times10^{-11}\times\frac{1.898\times10^{27}}{(7.15\times10^{7})^2}$ or $g_J = 24.77\,\text{m s}^{-2}$

So, value of 'g' on Jupiter's surface if 24.77 which is about 2.528 times of the acceleration due to gravity on Earth's surface ($g_J = 2.528\ g_E$). Weight of astronaut can be found by:

$$W_A = mg_J$$

Putting values, we get: $W_A = 60\times24.77\,\text{N}$ or $W_A = 1486.2\,\text{N}$

While on Earth, he will only weigh 588.6 N.

Assignment 15.3

The gravitational field strength on surface of moon is $1.6\ \text{N kg}^{-1}$. The mass of moon is 7.3×10^{22} kg, what is its radius?

15.3 SATELLITES AND ORBITS

A satellite is any object that orbits the planet due to the force of gravity, maintaining a stable path around it.

A natural body orbiting a planet, dwarf planet, or minor planet, where the larger body's gravity dominates the system, is called a natural satellite. Six of the major planets possess natural satellites often termed as moons.

For Your Information

Space station is a space craft capable of supporting crew which is designed to remain in space for an extended period of time and to which other space-crafts can dock. International Space Station (ISS) is the largest satellite in the orbit; it can even be seen with the naked eye.

Artificial Satellites

Artificial satellites are objects intentionally placed into orbit around the Earth or other celestial bodies. Different orbits for artificial satellites are shown in Fig. 15.6. The first artificial satellite was launched in 1957, and since then, thousands have been sent for various purposes such as communication, military operations, and scientific research.

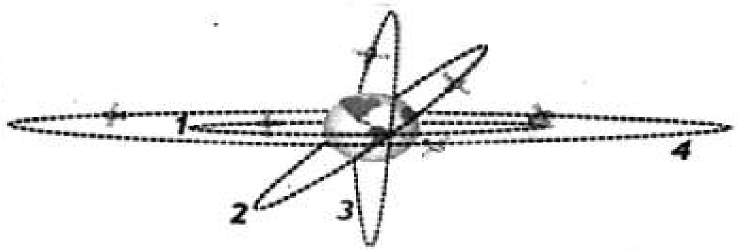


Figure 15.6: Different orbits for artificial satellites are shown with numbers. 1 shows plane of the equator, 2 shows inclined orbit, 3 shows the polar orbit, 4 shows the geostationary orbit.

These satellites orbit the Earth without the need for an engine, as they are held in place by the gravitational pull of the Earth. Engineers have developed different types of satellites, each serving a specific purpose or mission, such as: Communication satellites, Weather satellites, Navigation satellites etc.

Launching a satellite into orbit involves a complex and carefully planned process to ensure mission success. A satellite is put into orbit by moving it to high altitude and then accelerating it to a sufficiently high tangential speed with the help of space craft (e.g. rockets), as shown in Fig. 15.7. If the speed is too low the satellite will fall back to Earth. If the speed is too high, the satellite will either move in elliptical orbit or will escape out of the Earth's gravity, never to return (escape speed). However, if the speed is adjusted it will move in a circular orbit forever. Satellites are typically put into circular (or nearly circular) orbits, because such orbits require the least energy.

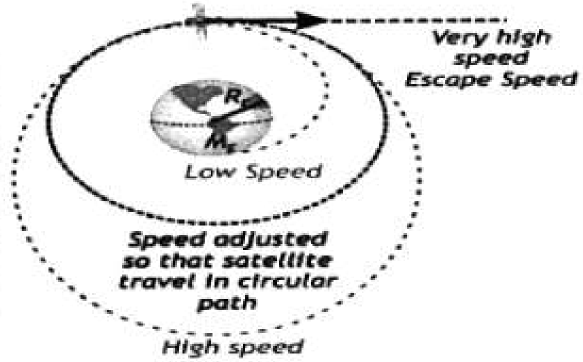


Figure 15.7: Artificial satellites launched at different speed.

Orbital Velocity

The orbital velocity of a satellite refers to the minimum velocity required for a satellite to orbit the Earth or another celestial body at a specific altitude.

When a satellite is moving with velocity 'v_o' in a circle of radius 'r' from the centre of earth, it has centripetal acceleration given by:

$$a_c = \frac{v_o^2}{r}$$

As this centripetal acceleration is supplied by gravity, i.e., $a_c = g$,

So, $$g = \frac{v_o^2}{r}$$

$$v_o^2 = g\,r$$

or $$v_o = \sqrt{g\,r}$$

By using $g = G\frac{M_E}{r^2}$, we get:

$$v_o = \sqrt{\frac{GM_E}{r}} \quad \text{———— (15.8)}$$

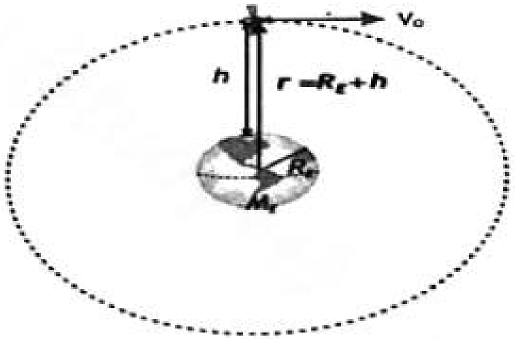


Figure 15.8: Satellite launched at orbital speed.

Hence orbital velocity also depends upon the mass of the larger body and the distance from the centre of the larger body to the centre of mass of the satellite.

Eq. (15.8) tells us that the satellite in the orbit moves faster when it is close to the gravitating body (i.e., Earth) and slower when it is further away.

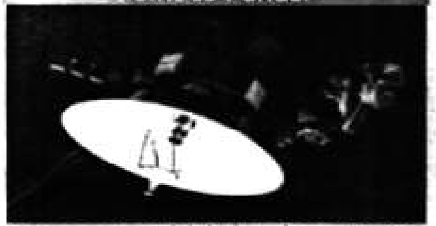
As the size of satellite is small as compared to the size of Earth and the distance in between therefore 'r' is taken as the sum of the radius of Earth 'R_E' and height 'h'.

Therefore, Eq. (15.9), can also be written as:

$$v_o = \sqrt{\frac{GM_E}{R_E + h}} \quad \text{——— (15.9)}$$

As the speed decreases due to height, we may have certain orbit such that the satellite covers a complete round trip in twenty-four hours and it appears stationary above certain fixed point.

Point to Ponder



Voyager 1, which was launched in 1977 alongside Voyager 2, holds the title of being the farthest manmade object. It has recently ventured into interstellar space, surpassing the distance between Earth and Pluto. These spacecrafts are equipped with a golden record containing various messages from Earth, such as music and speeches, intended for potential extraterrestrial beings to appreciate.

Example 15.4: The International Space Station (ISS) is the biggest structure ever placed in space (even visible in night sky with unaided eye) and serves as a space laboratory. It orbits at an average altitude of about 400 km above the Earth's surface in Low Earth Orbit (LEO). What is its orbital velocity?

Given: h = 400 km = 400,000 m

To Find: v_o = ?

Solution: The equation for orbital velocity is:

$$v_o = \sqrt{\frac{GM_E}{R_E + h}}$$

Putting values, we get: $v_o = \sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{6.4\times10^{6}+400,000}}$ or $v_o = 7.66\ \text{km s}^{-1}$

At this speed in a span of 24 hours, the space station completes 16 orbits around Earth, experiencing 16 sunrises and sunsets along the way.

Example 15.5: The mass of the Sun is 1.99×10^{30} kg, and the radius of the Earth's orbit around the Sun is 1.5×10^{11} m. What is the orbital velocity of the Earth?

Given: $M_s = 1.99 \times 10^{30}$ kg $r = 1.5\times10^{11}$ m

To Find: v_o = ?

Solution: The equation for orbital velocity is:

$$v_o = \sqrt{G\frac{M_S}{r}}$$

Putting values, we get: $v_o = \sqrt{6.67\times10^{-11}\times\frac{1.99\times10^{30}}{1.5\times10^{11}}}$ or $v_o = 30\ \text{km s}^{-1}$

This is indeed very high speed; it means that we all sitting stationary on surface of Earth are actually moving at 30 km s^{-1} around the sun.

Assignment 15.4

Calculate the orbital speed of satellite orbiting the Earth at an altitude equal to Earth's radius.

15.4 GEOSTATIONARY SATELLITES

At certain distance from the centre of Earth a satellite would take exactly 24 hours to circle the Earth. Such satellite would remain stationary above some point on Earth. These satellites are called geostationary (or geosynchronous satellites) and the orbit of these satellites is called geostationary orbit, as shown in Fig. 15.9.

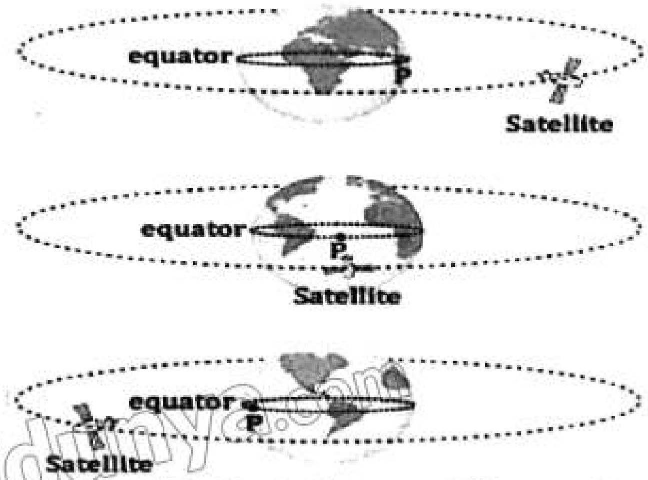


Figure 15.9: Geostationary satellite remains stationary above point 'P' on the surface of Earth.

Consider a satellite of mass m revolving in a geostationary orbit with velocity 'v_o' from Earth of mass 'M_E'. Let 'r' be the distance between the centre of Earth and the centre of satellite. Then the orbital velocity of satellite is given by:

$$v_o = \sqrt{\frac{GM_E}{r}}$$

For a satellite revolving in a circular geostationary orbit of radius 'r' the distance covered by satellite is the circumference of circle ($2\pi r$) and time taken is the time period 'T' of satellite, then velocity 'v_o' is given by:

$$v_o = \frac{S}{t} = \frac{2\pi r}{T}$$

Comparing the above two equations for velocity 'v_o', we get: $\frac{2\pi r}{T} = \sqrt{\frac{GM_E}{r}}$

Squaring both sides and rearranging for 'r', we get: $r^3 = \frac{GM_E T^2}{4\pi^2}$

Taking cube root on both sides, we get:

$$r = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3} \qquad \text{———— (15.10)}$$

Eq. (15.10) gives the orbital radius of geostationary satellite.

Calculation of orbital radius for geostationary satellite

Since $G = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$, $M_E = 6\times10^{24}$ kg, T = 86400 s and $\pi = 3.14$ by substituting these values in Eq. (15.10), we get:

$$r = \left(\frac{6.67\times10^{-11}\times6\times10^{24}\times(86400)^2}{4\times(3.14)^2}\right)^{1/3}$$

$r = 4.23\times10^{7}$ m or $r = 4.23\times10^{4}$ km

This is the orbital radius as measured from the centre of Earth.

Calculation of orbital speed of geostationary satellite

The equation for orbital speed is: $v_o = \sqrt{\dfrac{GM_E}{r}}$

Since G = 6.67×10^{-11} N m^2 kg^{-2}, $M_E = 5.98\times10^{24}$ kg and r = 4.23×10^{7} m, by substituting values, we get:

$$v_o = \sqrt{6.67\times10^{-11}\times\frac{6\times10^{24}}{4.23\times10^{7}}} = 3.07\times10^{3}\ \text{m s}^{-1}$$

Point to Ponder

A satellite that stays fixed relative to the Moon's surface, known as a lunar-stationary satellite, cannot maintain a stable orbit due to the Moon's Hill Sphere. The Hill Sphere is a region around a celestial body where its gravitational pull is stronger than that of a larger body, in this case, Earth. To have a satellite stationary relative to the Moon, its orbital period would have to match the Moon's rotation period of about 27.3 days. The required orbital radius for this 27.3-day period around the Moon is approximately 88,417 km. However, the Moon's Hill Sphere extends only up to around 60,000 km. This means that the calculated radius of 88,417 km falls outside the Moon's Hill Sphere, causing Earth's gravitational force to interfere and destabilize the satellite's orbit. As a result, a lunar-stationary satellite would not be able to sustain a stable orbit around the Moon.

Example 15.6: Venus has a mass of 4.867×10^{24} kg and has a period of 243 days. What would be the radius of a synchronous satellite for this planet?

Given: $M_v = 4.867\times10^{24}$ kg T = 243 days = 2.093×10^{7} s

To Find: r = ?

Solution: **The equation for radius of synchronous satellite for Venus is:**

$$r = \left(\frac{GM_v T^2}{4\pi^2}\right)^{1/3}$$

Putting values, we get: $$r = \left(\frac{6.67\times10^{-11}\times4.867\times10^{24}\times(2.093\times10^{7})^2}{4\times(3.14)^2}\right)^{1/3}$$

or $r = 1.53\times10^{9}$ m = 1.53×10^{6} km

Venus, due to its incredibly slow rotation, cannot have a stable geostationary orbit at 1,530,000 km (or even close to that distance). Due to Venus's lower mass compared to Earth, its Hill sphere is significantly smaller. At a distance of 1,530,000 km, the satellite would likely be outside Venus's Hill sphere and more influenced by the Sun's gravity. This would cause the satellite's orbit to become unstable, and it wouldn't remain stationary over a fixed point on Venus.

Therefore, while the mathematical formula provides a solution, it doesn't represent a feasible scenario for a synchronous satellite around Venus.

Assignment 15.5

Calculate the height (from surface of Earth) of a satellite in geostationary orbit.

15.5 GRAVITATIONAL POTENTIAL

The formula $\Delta P.E = mgh$ is accurate for potential energy near the Earth's surface where gravity is constant. However, as we move away from the surface, gravity weakens, making the equation invalid. Instead, we must use an expression based on Newton's law of universal gravitation.

Secondly, to calculate gravitational potential energy, we need to establish a zero-reference point, often the Earth's surface. This choice results in negative potential energy for any finite distance 'r' because potential energy decreases as objects move closer to the zero-reference point and increases as they move apart.

Consider the Fig. 15.10 in which a body of mass 'm' is placed at surface of Earth having distance from the centre of the Earth 'R_E' (equal to the radius of Earth). As mass of Earth is 'M_E' and to displace the body from point '1' to 'n', we divide the whole distance into number of small distances each of magnitude 'Δr', such that the force during each interval remains constant.

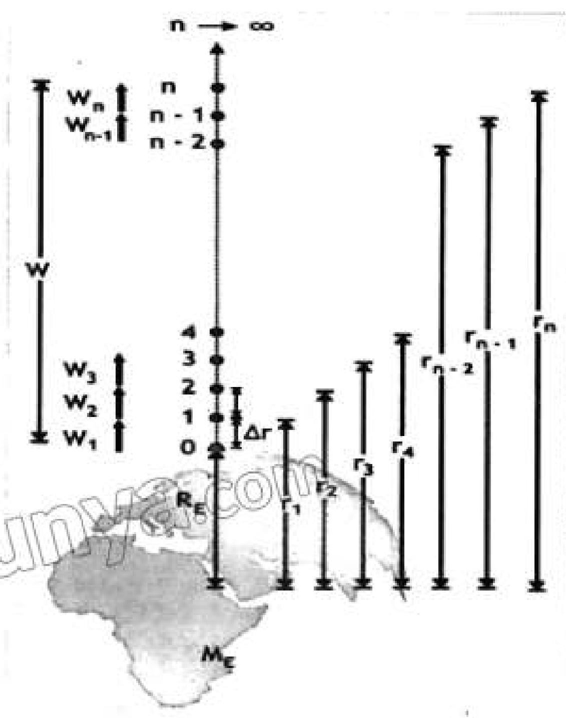


Figure 15.10: Gravitational Potential.

By work and potential energy principle, we get:

$$\Delta P.E = W_{net} \quad \text{______ (1)}$$

As we go up from the surface of Earth, the gravitational force decrease. So, for variable force, we can get the total work done by summing up all the individual work done, i.e.,

$$W_{net} = W_1 + W_2 + W_3 + \ldots\ldots + W_{n-1} + W_n \quad \text{______ (2)}$$

The work done from point '0' to point '1' is 'W_1', which can be written as:

$$W_1 = F_{av}.\ \Delta r \qquad \text{or} \qquad W_1 = F_{av} \Delta r \cos\theta$$

Here $\theta = 180°$ and $\cos 180° = -1$

Therefore, $\quad W_1 = -F_{av}\ \Delta r \quad$ ______ (3)

The average force from point '0' to point '1' is F_{av}, which can be written from Newton's law of universal gravitation as:

$$F_{av} = \frac{G M_E m}{r_{av}^2} \quad \text{______ (4)}$$

Now the average distance 'r_{av}' is neither 'R_E' nor 'r_1', it can be calculated as:

Calculation of r_{av}:

The average distance r_{av} is $r_{av} = \frac{R_E + r_1}{2}$ ______ (5)

From the Fig. 15.10: $\Delta r = r_1 - R_E$ or $r_1 = R_E + \Delta r$ ______ (6)

Putting Eq. (6) in Eq. (5), we get:

$$r_{av} = \frac{R_E + R_E + \Delta r}{2} \quad \text{or} \quad r_{av} = \frac{2R_E + \Delta r}{2}$$

Therefore. $r_{av} = R_E + \frac{\Delta r}{2}$ ______ (7)

Calculating $(r_{av})^2$:

Squaring both sides of Eq. (7), we get: $r_{av}^2 = \left(R_1 + \frac{\Delta r}{2}\right)^2$

Therefore, $r_{av}^2 = \left(R_E^2 + \frac{(\Delta r)^2}{4} + \frac{2R_E\Delta r}{2}\right)$

Since 'Δr' is very very small, the square of the term will be so small that it approaches to ZERO, compared to 'R_E' and 'r_1'. Therefore, we can neglect the term which include $(\Delta r)^2$, as it will have extremely minor effect on overall calculation, hence:

$$r_{av}^2 = R_E^2 + R_E\Delta r \quad \text{______ (8)}$$

Putting value of Δr from equation (6) in equation (8), we get:

$$r_{av}^2 = R_E^2 + R_E(r_1 - R_E) \quad \text{or} \quad r_{av}^2 = R_E^2 + R_E r_1 - R_E^2$$

So $r_{av}^2 = R_E r_1$ ______ (9)

Putting Eq. (9) in Eq. (4), we get: $F_{av} = \frac{GM_E m}{R_E r_1}$ ______ (10)

From the Fig. 15.10, $\Delta r = r_1 - R_E$ ______ (11)

Putting values from Eq. (10) and Eq. (11) in Eq. (3), we get:

$$W_1 = -\frac{GM_E m}{R_E r_1}(r_1 - R_E)$$

Hence $W_1 = -GM_E m\left(\frac{1}{R_E} - \frac{1}{r_1}\right)$ ______ (12)

This is the value of work in which a force is supposed to remain constant during separation 'Δr', similarly,

$$W_2 = -GM_E m\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad \text{______ (13)}$$

and $$W_3 = -GM_E m\left(\frac{1}{r_2}-\frac{1}{r_3}\right) \quad \text{------ (14)}$$

Similarly, the work done in last two steps is given by:

$$W_{n-1} = -GM_E m\left(\frac{1}{r_{n-2}}-\frac{1}{r_{n-1}}\right) \quad \text{------ (15)}$$

and $$W_n = -GM_E m\left(\frac{1}{r_{n-1}}-\frac{1}{r_n}\right) \quad \text{------ (16)}$$

The total work done in moving a body from point '0' to point 'n' can be obtained by adding all the individual work done, that is, by putting Eq. (12) to Eq. (16) in Eq. (2), we get:

$$W_{net} = -GM_E m\left(\frac{1}{R_E}-\frac{1}{r_1}\right) - GM_E m\left(\frac{1}{r_1}-\frac{1}{r_2}\right) - GM_E m\left(\frac{1}{r_2}-\frac{1}{r_3}\right) + \ldots\ldots$$

$$\ldots\ldots - GM_E m\left(\frac{1}{r_{n-2}}-\frac{1}{r_{n-1}}\right) - GM_E m\left(\frac{1}{r_{n-1}}-\frac{1}{r_n}\right)$$

or $$W_{net} = -GM_E m\left(\frac{1}{R_E}-\frac{1}{r_1}+\frac{1}{r_1}-\frac{1}{r_2}+\frac{1}{r_2}-\frac{1}{r_3}+\ldots\ldots\ldots\ldots+\frac{1}{r_{n-2}}-\frac{1}{r_{n-1}}+\frac{1}{r_{n-1}}-\frac{1}{r_n}\right)$$

Therefore; $$W_{net} = -GM_E m\left(\frac{1}{R_E}-\frac{1}{r_n}\right) \quad \text{------ (17)}$$

Putting Eq. (17) in Eq. (1), we get:

$$\Delta P.E = -GM_E m\left(\frac{1}{R_E}-\frac{1}{r_n}\right) \quad \text{------ (18)}$$

Now to exclude the choice of setting a reference point for calculation, we set point 'n' at infinity, such that we define the gravitational potential energy 'U' as the work done in bringing a mass 'm' from infinity to the surface of Earth.

When point 'n' is taken at infinity, then '$r_n = \infty$', and Eq. (18) becomes:

$$U = -GM_E m\left(\frac{1}{R_E}-\frac{1}{\infty}\right) \qquad \text{as} \qquad \frac{1}{\infty}=0$$

Therefore, the general expression for the gravitational potential energy 'U' of a body situated on the surface of Earth at distance 'r' from the centre of Earth is given by:

$$U = -\frac{GM_E m}{r} \quad \text{------ (15.9)}$$

Eq. (15.9) is used when gravitational force is not constant. The variation of gravitational potential energy 'U' as a function of distance 'r' is shown in the Fig. 15.11. The absolute potential energy is defined as:

The amount of work done in moving a body from Earth's surface to a point at infinite distance where the value of g is negligible.

The gravitational potential (represented by ϕ or V) at a point is the work done per unit mass in bringing a small test mass from infinity to that point. The gravitational potential 'V' is therefore the gravitational potential energy 'U' per unit mass 'm', mathematically:

$$V = \frac{U}{m} \quad \text{\_\_\_\_\_\_ (15.10)}$$

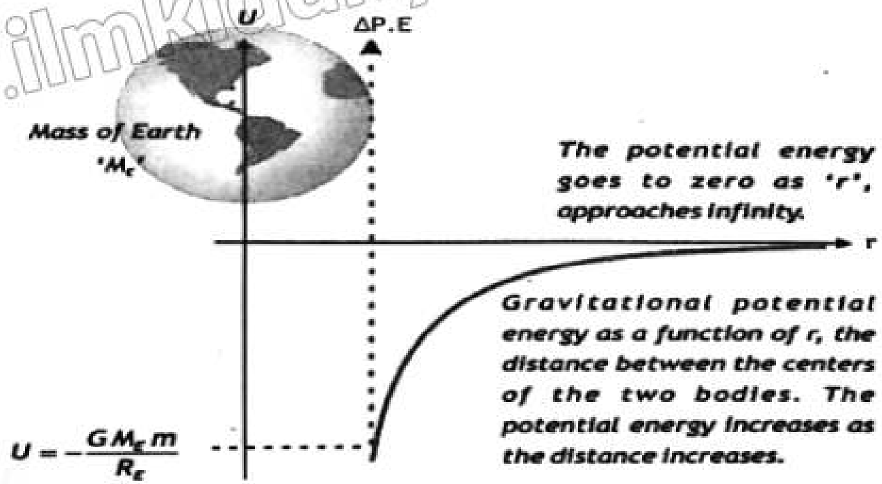


Figure 15.11: Variation of the gravitational field and potential, as a function of the distance from the center.

The expression for gravitational potential 'V' at a point 'P' at distance 'r' from the centre of Earth, can therefore be obtained by putting Eq. (15.9) in Eq. (15.10), such that:

$$V = -\frac{GM_E}{r} \quad \text{\_\_\_\_\_\_ (15.11)}$$

Calculation of Gravitational Potential at the Surface of Earth:

Eq. (15.11) can be used to determine the gravitational potential at the surface of Earth as it does not depend upon the mass of the object. As gravitational constant G is 6.67×10^{-11} N m^2 kg^{-2}, mass of Earth M_E is 6×10^{24} kg and radius of Earth R_E is 6.4×10^6 m, therefore:

$$V = -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^6}$$

or $\quad V = -62.3 \times 10^6 \text{ N m kg}^{-1} = -62.3 \text{ MJ kg}^{-1}$

Although the equations are derived for Earth, but these are equally valid for other gravitating objects.

Example 15.7: What is the gravitational potential energy and gravitational potential with respect to the Sun at the position of the Earth? The mass of the Sun is 1.99×10^{30} kg and the mass of the Earth is 6×10^{24} kg. The mean Earth-to-Sun distance is 1.5×10^{11} m.

Given: $M_S = 1.99 \times 10^{30}$ kg $\quad M_E = 6 \times 10^{24}$ kg $\quad r = 1.5 \times 10^{11}$ m
To Find: U = ? $\quad$ V = ?
Solution: Gravitational potential energy formula for sun and Earth can be written as:

$$U = -\frac{G M_S M_E}{r}$$

Putting values from given data, we get:

$$U = -\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 6 \times 10^{24}}{1.5 \times 10^{11}}$$

Therefore, $U = -5.29 \times 10^{33}\ \text{N m} = -5.29 \times 10^{33}\ \text{J}$

Gravitational potential formula for sun and Earth can be written as:

$$V = -\frac{G M_S}{r}$$

Putting values from given data, we get:

$$V = -\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{1.5 \times 10^{11}}$$

or $V = -8.85 \times 10^{8}\ \text{N m kg}^{-1} = -885\ \text{MJ kg}^{-1}$

Assignment 15.6

Calculate the value of gravitational potential at 1000 km, 50,000 km, and 100,000 km from the surface of Earth. Compare the values obtained with potential energy formula '$E_P = mgh$'.

SUMMARY

- **Newton's Law of Universal Gravitation:** Every object in the universe attracts every other object with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.
- **Satellite:** A satellite is any object that orbits another object due to the force of gravity, maintaining a stable path around it.
- **Orbital velocity:** In circular orbit a satellite has a constant tangential speed called orbital velocity.
- **Geostationary satellite:** A geostationary satellite, also known as a geosynchronous equatorial orbit (GEO) satellite, is a type of satellite that orbits the Earth directly above the equator at an altitude where its orbital period matches the Earth's rotation period. This results in the satellite appearing stationary relative to a fixed point on the Earth's surface.
- **Absolute gravitational potential energy:** The potential energy possessed by a body at a certain height in a gravitational field with respect to reference point of zero potential is known as absolute potential energy.
- **Gravitational potential:** The gravitational potential is the gravitational potential energy per unit mass.

Formula Sheet

$g = G\frac{m}{r^2}$ $\qquad g_h = \frac{g R_E^2}{(R_E + h)^2}$ $\qquad v_o = \sqrt{\frac{G M_E}{R_E + h}}$

$$r = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3} \qquad U = -\frac{GM_E m}{R_E} \qquad V = -\frac{GM_E}{R_E}$$

$$F_g = G\frac{m_1 \times m_2}{r^2}$$

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) Two identical balls of masses 1 kg each having distance of 1 m between their centres then gravitational force between them is:

A. 667×10^{-9} N B. 6.67×10^{-11} N
C. 667×10^{11} N D. 6.67×10^{-13} N

2) Gravitational force between two objects is 'F'. If masses of bodies are doubled and distance between their centres is reduced to half, then gravitational force is:

A. F B. 4 F C. F/4 D. 16 F

3) The value of 'g' at height of 1500 km above the surface of Earth in m s^{-2} is:

A. 0 B. 6.4 C. 9.8 D. 12.2

4) If we consider Earth as perfect sphere and ignore the presence of air resistance. What will be the orbital velocity to launch a satellite in circular orbit just above the surface at $r = R_E = 6.4 \times 10^6$ m:

A. 0 km s^{-1} B. 1.19 km s^{-1} C. 3.07 km s^{-1} D. 7.9 km s^{-1}

5) When a satellite is put into a higher circular orbit, its kinetic energy:

A. increases B. decreases C. is zero D. remains the same.

6) The orbital velocity of geostationary satellite is

A. 0 km s^{-1} B. 1.19 km s^{-1} C. 3.07 km s^{-1} D. 7.9 km s^{-1}

7) The minimum number of geostationary satellites for the complete global coverage is

A. 1 B. 2 C. 3 D. 4

8) When an object moves away from a massive body, its gravitational potential energy:

A. increases B. decreases C. remains constant D. becomes zero

9) The value of gravitational potential at an altitude of 35700 km above the Earth's surface, where communication satellites orbit the Earth is approximately:

A. − 94.7 M J kg^{-1} B. − 62.3 M J kg^{-1}
C. + 947.11 J kg^{-1} D. − 947.11 J kg^{-1}

Short Questions

1) Why can gravitation not account for the formation of molecules?
2) Why don't two books on your desk attract each other gravitationally, despite Newton's law of gravitation?
3) Why does an apple fall towards the Earth due to gravity, while the Earth doesn't move towards the apple?
4) Can gravitational field strength be negative? Explain.
5) What factors determine the strength of the gravitational field around a planet?
6) If two planets have the same mass but different radii. How would their gravitational field strengths compare?
7) Why satellites in higher orbits have lower orbital velocities?
8) How does the mass of the Earth affect the orbital velocity required for a satellite to stay in orbit?
9) Is it possible for an object's gravitational potential energy to become negative? If so, what does this mean for the object's motion?
10) How the gravitational potential energy of two point masses is related to concept of gravitational potential?

Comprehensive Questions

1) What is force of gravity? State and explain Newton's law of gravitation. Also show that Newton's law of gravitation is consistent with Newton's 3rd law of motion.
2) What are gravitational field and gravitational field strength? Explain. Derive the formula for gravitational acceleration 'g' on the surface of Earth and discuss the variation of 'g' with altitude.
3) How do engineers calculate the required velocity for a satellite to be placed into a specific orbit? Can you describe the steps involved in determining the orbital parameters necessary for a successful satellite launch and deployment?
4) What are geostationary satellites? Calculate the orbital radius 'r', orbital speed 'v_o' and height 'h' above surface of Earth for the geostationary orbit.
5) If 'M_E' is the mass of Earth and 'r' is the distance of unit mass from the centre of Earth and 'G' is the universal gravitational constant. Prove that gravitational potential 'V' can be written as: $V = -\frac{G M_E}{r}$

Numerical Problems

1) Find the mass of the Earth by considering a scenario where a small object is placed on the Earth's surface such that distance between their centres is equal to the Earth's radius, i.e., 6.4×10^6 m. **(Ans: 6.4×10^{24} kg)**

2) Alpha Centauri is a binary star system with two stars: Alpha Centauri A and Alpha Centauri B. The mass of Alpha Centauri A is 2.19×10^{30} kg and Alpha Centauri B is 1.80×10^{30} kg, they attract each other with a force of 2.24×10^{25} N. What is the distance between the two stars?

(Ans: 3.43×10^{12} m)

3) What will be value of 'g' on an exoplanet (a planet outside the Solar System) whose mass is five times the mass of Earth and its radius is twice the radius of Earth? **(Ans: 12.25 m s^{-2})**

4) The Hubble Space Telescope is in a circular orbit 613 km above Earth's surface. The average radius of the Earth is 6.4×10^6 m and the mass of Earth is 6×10^{24} kg. (a) What is the speed of the telescope in its orbit? (b) What is the period of the telescope's orbit?

[Ans: (a) 7550 m s^{-1}, (b) 5807 s]

5) To launch a satellite around planet Mars at an altitude of 300 km above its surface. Mars has a mass of 6.42×10^{23} kg and has a radius of 3.39×10^6 m, calculate its orbital velocity.

(Ans: 3.4 km s^{-1})

6) An asteroid orbits the sun 8.35×10^{11} m from it. (a) How fast must the asteroid travel to maintain its circular orbit around the sun? (b) How long will it take the asteroid to orbit the sun? **(Ans: 1.26×10^4 m s^{-1}, 13.2 years)**

7) Mars has a mass of 6.42×10^{23} kg and has a period of 88,642 s. What would be the radius of a stationary satellite for this planet? **(Ans: 20,428 km)**

8) Calculate the potential energy of the Moon having mass 7.35×10^{22} kg, relative to Earth with mass 6×10^{24} kg if the distance between their centres is 3.94×10^5 km. **(Ans: -7.4×10^{28} J)**

9) A 50 kg weather satellites move in circular orbits about the Earth at an altitude of 1000 km. A similar 50 kg communication satellite is at an altitude of 37,000 km in circular orbits about the Earth. Calculate the difference in the gravitational potential energies of the two satellites in their respective orbits? **(Ans: 2.2 GJ)**

10) What is the change in gravitational potential energy of a 64.5 kg astronaut, lifted from Earth's surface into a circular orbit of altitude 4.40×10^2 km? **(Ans: 2.6×10^8 J)**